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Step-induced separation of a turbulent boundary layer in incompressible flow

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Measurements of the low-speed flow up a step of height equal to 1·75 times the initial boundary-layer thickness show that the flow satisfies Stratford's (1959) condition for rapid separation, the extra stress gradients being confined to the first one-eighth of the boundary-layer thickness. The increase in turbulence intensity up to separation is small, and attributable to low-frequency fluctuations in separation position. Townsend's (1962) criterion predicts the separation point fairly accurately. A simple expression is found for the additional pressure rise that can be withstood by a boundary layer already fairly near separation, which gives tolerable results at any point in the flow up a step.

1. Introduction

The flow up a step is so frequently mentioned as the prototype of rapidly separating flows that it is surprising that no measurements of the boundary-layer development in low-speed flow are available. Lighthill (1953) apparently did not know of any, and, when we started this work, we were unable to find any published since then: in fact, the most comprehensive study was that of Head & Rechenberg (1962), who measured the pressure coefficient and surface shear stress in the course of an experiment on the reliability of surface tubes. The well-known work of Chapman, Kuehn & Larson (1958) was confined to pressure measurements and shadowgraph observations, and most of it was done at supersonic speeds, where the problem becomes one of shock-wave boundary-layer interaction. Partly because the shock wave dominates the flow, and partly because there is a direct connexion between flow angle and static pressure, the supersonic case does not throw much light on the low-speed flow or on low-speed separation in general.

The Stratford (1954, 1959) and Townsend (1960, 1962) sequence of papers on the prediction of separation makes the simplifying approximation that the change of total pressure along streamlines in the outer part of the flow is negligible in the region of strong adverse pressure gradient preceding separation, i.e. that the effect of the stress gradients in the outer part of the initial boundary layer is negligible. Modified mixing-length theory and the assumption that the stress gradient in the inner part of the flow (the 'equilibrium layer') is independent of y, lead to the result that, in the equilibrium layer, the departure of U/u_τ from the value in a constant-stress layer, $\{\log(u_\tau y/\nu)+A\}/K$, is a certain universal function of $\alpha y/\tau_w$, where α is the stress gradient. For large enough values of $\alpha y/\tau_w$, this gives

$$U = \frac{2}{K_0}\left(\frac{\alpha y}{\rho}\right)^{\frac{1}{2}} + U_t,$$

where K_0 is about 0·50, U_t is independent of y and falls to practically zero at separation.

The usual test cases for methods of predicting separation are those of Newman (1951), whose boundary layer almost separated, and Schubauer & Klebanoff (1951). The former has a very long region of adverse pressure gradient so that it is difficult to distinguish the phenomena characteristic of separation from the delayed effects of the upstream history of the boundary layer. Although the latter closely represents a boundary layer passing from a region of zero pressure gradient into a region of strong adverse pressure gradient, the distance between the start of the pressure gradient and the separation point is 40 times the initial thickness of the boundary layer or nearly half the distance from the leading edge to the start of the pressure gradient, so that the Stratford approximation is not very well satisfied. A set of measurements in a rapidly separating boundary layer is therefore a useful test case for the Stratford–Townsend method.

The immediate reason for our own interest in the flow up a step is as part of a study of equilibrium turbulent boundary layers and their response to perturbations. A change in pressure-gradient parameter from one (constant) value to a slightly different (constant) value is a weak perturbation, and step-induced separation of a boundary layer initially in equilibrium is about the strongest perturbation that can be imagined, and also the simplest useful case of separation that can be investigated. The experiment reported here, on a boundary layer initially in zero pressure gradient, was a pilot study for this work, and consisted principally of mean-flow measurements up to the separation point: no velocity profiles were measured aft of separation, since the chief object in making fundamental studies of separation is to learn how to avoid it. One of the incidental results of the work is the discovery of a very simple criterion of 'nearness to separation' which should be rather useful in designing bodies where the boundary layer remains attached.

2. Apparatus and procedure

The measurements were made in the boundary layer on the wall of a 15 in. × 10 in. open-circuit tunnel, at a speed of 30 ft./s in the empty working section. The Reynolds number $U_1\delta_1/\nu$ was 3100. The free-stream turbulence level was about 0·2 %. The steps were mounted on the 15 in. side of the tunnel. The measuring instruments were attached to $3\frac{1}{2}$ in. discs inserted in the tunnel wall at about 56 in. from the downstream end of the contraction; the undisturbed boundary layer was about 1·3 in. thick. For convenience, most of the measurements were made by keeping the instruments fixed and *moving the step*, a time honoured procedure in tests at supersonic speeds, chiefly, it must be admitted, at higher Reynolds numbers and lower rates of boundary-layer growth than in the present experiment. It is immediately clear that the difference between the results obtained by moving the step, and those obtained by moving the instruments, will be small if the distance through which the step is moved is small compared with its distance from the origin of the boundary layer, and, in fact, the difference between the pressure distribution measured at up to 18 in. from the step was almost entirely attributable to the rather large static pressure gradient in the empty tunnel, about $-0{\cdot}002\,.\,\frac{1}{2}\rho U_{\text{ref}}^2$ per in. where U_{ref} is the speed in the empty tunnel at the position of the instruments. Physically, the consequence of moving the step is to conceal the fact that the boundary layer would grow appreciably in the length of the region of retardation even if the step were not there. Thus, the effect of the stress gradients (of which $\partial\tau/\partial y$ is the largest) in the initial boundary layer is eliminated from the results, and this has the advantage that the effect of the extra stress gradients set up by the retardation can be seen more readily. The main assumption of the Stratford–Townsend theory, that the initial stress gradients are negligible, has therefore been made in the experiment as well: as mentioned above it is well satisfied.

A more important source of error in experiments on separation is lack of two-dimensionality. The separation line was found to be straight over at least the central two-thirds of the span of the step, but the only evidence that convergence of the flow, due to the growth of boundary layers on the side walls, was also negligible is that given by figure 8. This figure shows the conservation of total pressure along streamlines in the outer part of the flow, a direct consequence of the unimportance, or, in the present experiment, the suppression, of the initial stress gradients. Since the streamlines were calculated from the two-dimensional continuity equation, any convergence would have led to apparent consistent changes in total pressure.

Pressures were measured with a null-reading micro-manometer (Bradshaw 1964), and the accuracy of reading can be judged from the information that the maximum pressure difference measured in figure 9 was about 0·003 in. water. Pressures were made dimensionless by dividing by the dynamic pressure in the empty tunnel.

Total-pressure and static-pressure profiles were measured with a flat Pitot tube 0·01 in. high, a Pitot tube 0·03 in. in diameter used where the flow direction was uncertain, and a disc static probe mounted in the (x, y)-plane. No corrections have been made for the effects of turbulence.

Surface shear stress was measured with a Preston tube 0·115 in. in diameter, since the pressure differences obtained with a sublayer fence were unmeasurably small. Head & Rechenberg's (1962) calibration was used. Near the separation point, the flow near the surface will be reversed for part of the time; since the Preston tube measures total pressure when the flow is attached, but something nearer static pressure when the flow is reversed, it will indicate too high a value of the mean shear stress. We have used a probe like a sublayer fence of span roughly equal to its height (figure 2) to investigate the region of separation on the 2·25 in. step only. Since a height of about 0·1 in. was needed to give a measurable pressure difference, the instrument is liable to indicate separation too far downstream. The 'fence' has not been calibrated accurately, but a rough calibration derived by comparing the readings with Preston tube readings at the large values of x' has been used to obtain values of surface shear stress near separation. Separation was indicated at about 2·6 in. upstream of the step, or 2·7 in. if a rough correction is made for the effects of finite height of the fence. Velocity traverses made with the 0·01 in. high Pitot tube indicate separation at about 2·5 in., but the figure of 2·7 in. is the most reliable. The difference between the two is only about 0·15 of the initial boundary-layer thickness. In experiments at higher speeds, a conventional sublayer fence (Head & Rechenberg 1962) should be adequate.

The measurements of pressure gradient shown in figure 9 were obtained from the difference in pressure between two static pressure tappings 0·22 in. apart, and are therefore averaged over this distance; the error should be negligible.

In general, the results of this experiment are rather more scattered than normal boundary-layer measurements, partly because of the unsteadiness of any separating flow and partly because such small pressure differences had to be measured. The scatter in figure 4(*b*) for ($x' \geqslant 4$ in.) and in figure 8(*a*) (for $\psi \geqslant 0{\cdot}2$ in.) is typical. Measurements near the separation point are necessarily unreliable.

3. Results and discussion

3.1. *Surface pressure and surface shear stress*

A few measurements of surface pressure coefficient and surface shear-stress coefficient, both based on the dynamic pressure in the empty tunnel $\frac{1}{2}\rho U_{\mathrm{ref}}^2$ were made with different ratios of step height to boundary-layer thickness (figures 1 and 2; note that x' is measured *upstream* from the step). If this ratio is large enough, the pressure distribution is expected to scale on step height alone. Lighthill's theory applies to this case, and it was hoped to obtain a reasonable approximation to it with a step height of only twice the boundary-layer thickness on the argument that the effect of the total-pressure deficit in the outer part of the boundary layer should be negligible. In fact, the pressure and surface shear-stress distributions very nearly scale on boundary-layer thickness, irrespective of step height, if the x-origin is shifted upstream by 1·7 step heights in each case. No deep significance is claimed for this, but it does mean that only one test case for calculation methods can be extracted from the results. Also, any comparison with Lighthill's theory would be futile. Larger ratios of step height to boundary-layer thickness could only have been attained by reducing the boundary-layer thickness and Reynolds number to unacceptably low values, or by using a step uncomfortably large for the tunnel. The only region of the pressure distribution that is of any immediate qualitative interest is the region near the separation point. It is noticeable that the pressure gradient decreases considerably just before separation because of the effect of the increasing displacement thickness on the external flow; the same effect occurs in the measurements of Schubauer & Klebanoff (1951) on an aerofoil and is commented on at length by Townsend (1962). After separation, the pressure gradient continues to decrease and actually becomes negative (this is more clearly seen in figure 9); very near the step the pressure starts to increase rapidly again. This behaviour is consistent with the undoubted presence of a layer of reversed flow, emanating from the first reattachment point part way up the step, turning abruptly at the bottom of the step and then being gradually re-entrained into the separated shear layer. The velocity of circulation, judging from the pressure rise at the foot of the step, is about a quarter of the reference velocity in the case of the 2·25 in. step.

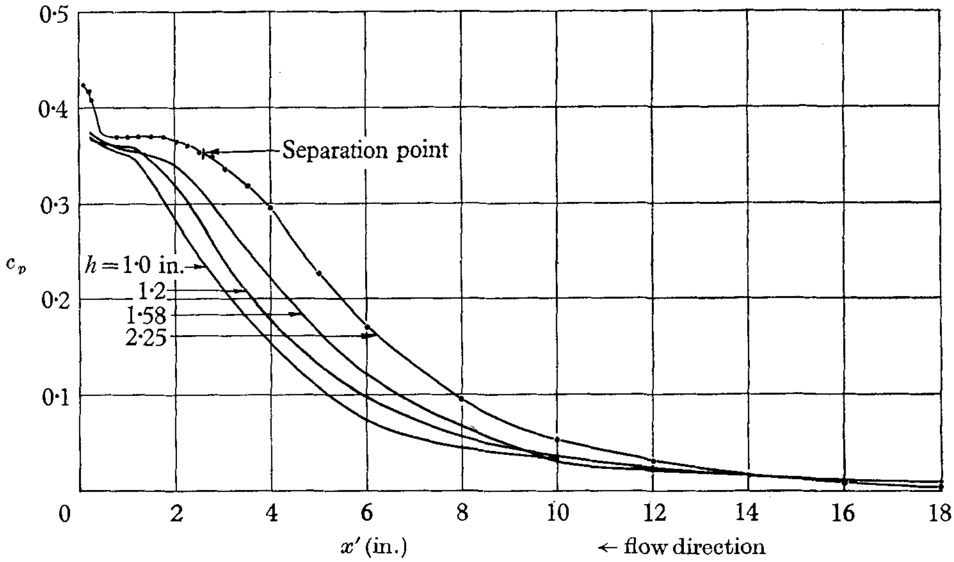


FIGURE 1. Surface pressure distributions for different step heights. The displacement thickness of the initial boundary layer is 0·194 in.

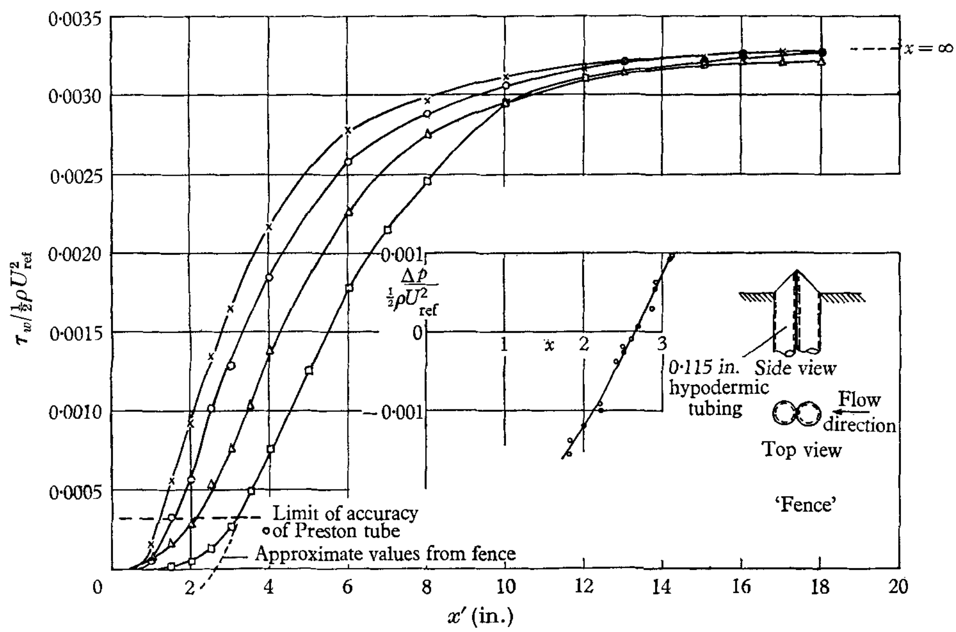


FIGURE 2. Surface shear-stress distributions for different step heights measured with a 0·115 in. Preston tube (Head & Rechenberg 1962). ×, h = 1·0; ○, h = 1·2; △, h = 1·58; □, h = 2·25. The inset shows the 'fence' and 'fence' readings taken near separation, h = 2·25 in.

The surface shear-stress distribution is likewise what might have been expected: the smallest values measured with the Preston tube are untrustworthy, but the 'fence' measurements, for the 2·25 in. step only, show that the surface shear stress decreases through zero almost linearly, another indication that there is a strong reversed flow in the separated region. In fact, the surface shear stress decreases linearly all the way from $r = 6$ in., where it is still as high as half its initial value, but this does *not* seem to be a general characteristic of separating

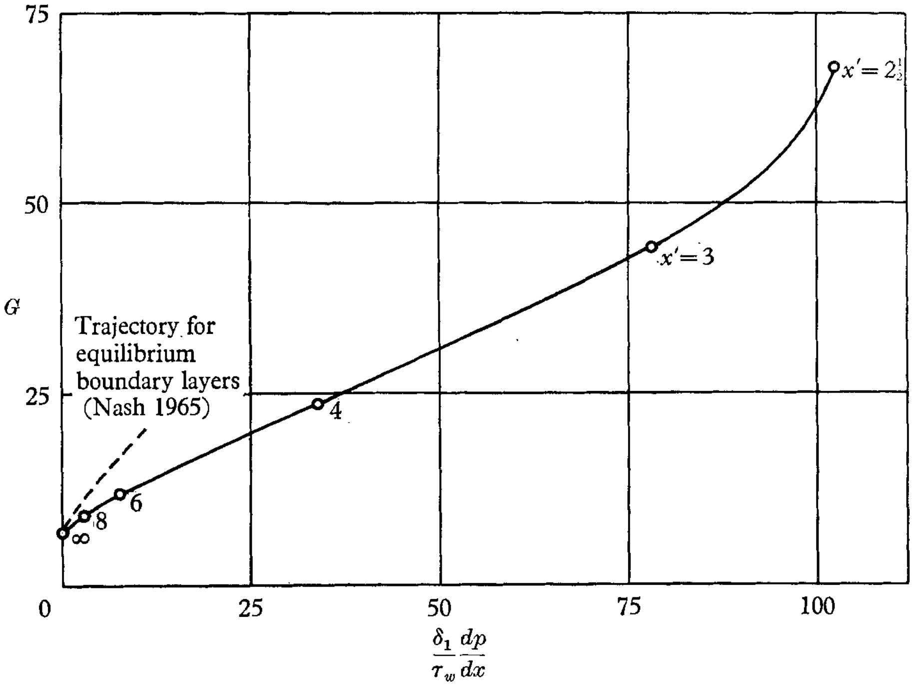


FIGURE 3. Variation of velocity-defect parameter G with pressure gradient.

flows. The formulae of Ludwieg & Tillman (1950), and others, greatly overestimate the surface shear stress almost from the start of the adverse pressure gradient; they are based on data from boundary layers more nearly in equilibrium. They give reasonable values for the surface shear stress in Schubauer & Klebanoff's experiment, but the maximum value of $(\delta_1/\frac{1}{2}\rho U_1^2)(dp/dx)$ in that experiment was less than half the maximum in the present work.

The velocity-defect profile parameter,

$$G = \int_0^\infty (U_1 - U)^2 dy/u_\tau \int_0^\infty (U_1 - U)\,dy,$$

is plotted against the pressure-gradient parameter $(\delta_1/\tau_w)\,dp/dx$ in figure 3, compared with the locus of all equilibrium boundary layers according to Nash (1965). Clearly the boundary layer is very far from local equilibrium.

3.2. *Measurements within the shear layer*

The 2·25 in. step was used for all the measurements of boundary-layer characteristics. The velocity profiles, in figure 4(*a*) are not particularly informative about the behaviour of the boundary layer because the static-pressure gradients normal to the surface (figure 7) are considerable—compare the values of U/U_{ref} at the total-pressure boundary with the values of $(p-p_w)/\frac{1}{2}\rho U_{\text{ref}}^2$ in figure 7—so that measurements of δ_1, δ_2 and H are rather meaningless. (If H were calculated

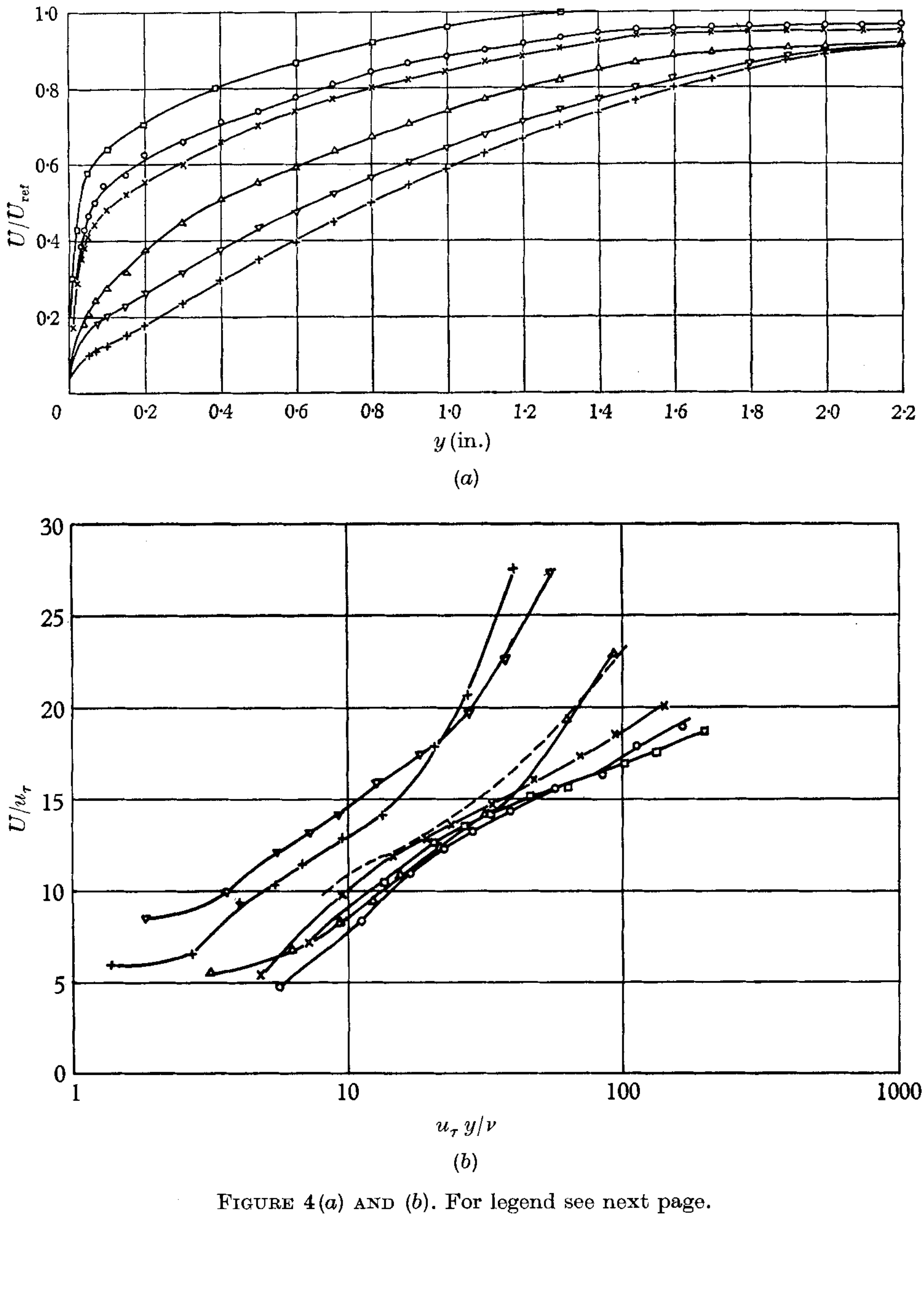
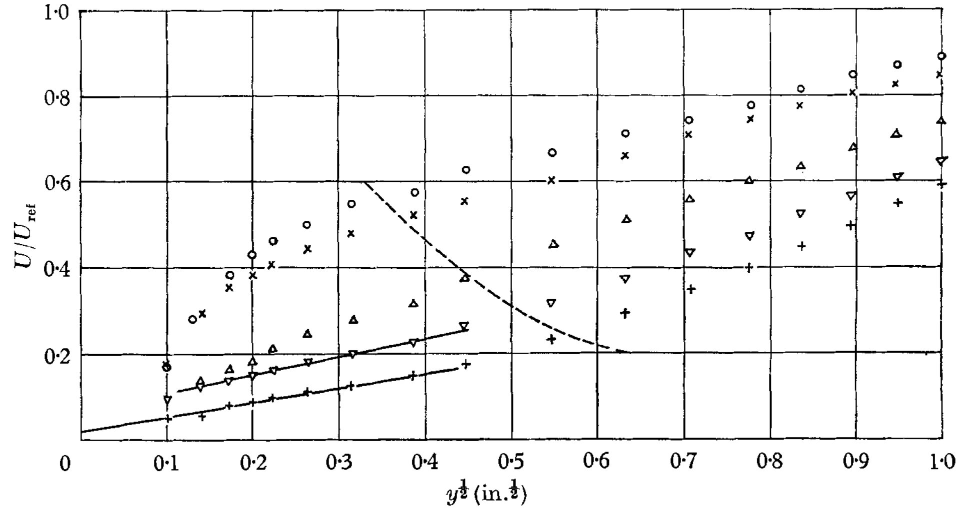


FIGURE 4(*a*) AND (*b*). For legend see next page.



(*c*)

FIGURE 4. Mean velocity profiles, $h = 2{\cdot}25$ in. (*a*) Linear plot. (*b*) Logarithmic plot; – – – –, $(\nu/u_\tau^3\rho)\,\partial\tau/\partial y = 0{\cdot}1$ (Mellor 1966) for comparison with $x' = 4$. (*c*) Square-root plot; – – –, locus of points where $\psi = 0{\cdot}05$ (approximately the edge of the equilibrium layer).

	x'	δ_1	H	$\tau_{\max}/\tau_w$
□	∞	0·194	1·357	—
○	8	0·272	1·462	1·34
×	6	0·310	1·550	1·84
△	4	0·457	1·858	4·36
▽	3	0·657	2·034	12·9
+	$2\frac{1}{2}$	0·781	2·381	—

from measurements of total pressure rather than dynamic pressure the values would be *lower* so the difference between the two curves in figure 3 would be even larger). The use of the boundary-layer approximation implies the assumption that the static pressure is constant across the shear layer. The velocity profiles in the inner part of the layer are plotted against $\log y$ in figure 4(*b*) and against $y^{\frac{1}{2}}$ in figure 4(*c*), the streamlines are shown in figure 6, and figures 8(*a*) and (*b*) show the total pressure profiles, referred to the static pressure in the empty tunnel, plotted against the stream function ψ. It should be noted that the measurements of static pressure (figure 7) for $y < 0{\cdot}4$ in. are implausible, although not to a sufficient degree to affect the velocity profiles significantly; a more nearly linear approach to the static pressure at the surface is expected, and the effect of turbulence on the static tube may be responsible. The surface pressures were, of course, measured at tappings in the wall of the tunnel.

It is seen from figure 8(*b*) that the effect of the *extra* shear-stress gradients caused by the presence of the step is confined to the region $\psi < 0{\cdot}05$ in. (Note once more that the effect of the initial stress gradients has been eliminated by moving the step. The three extra points in figure 8(*a*), which are to be compared with the circled points to the left of each, show the losses in total pressure, due to the initial stresses, which would have occurred between $x' = 8$ in. and $x' = 2\frac{1}{2}$ in.). As can be seen from figure 6, $\psi < 0{\cdot}05$ in. corresponds to about the first eighth of the boundary-layer thickness; the fraction should be almost independent of Reynolds number. Except in this wall equilibrium layer, the shear-stress profile is expected to remain almost the same as in the unperturbed boundary layer (see also figure 2 of Townsend (1962.) The region of adverse pressure gradient is far too short for the shear-producing eddies in the outer part

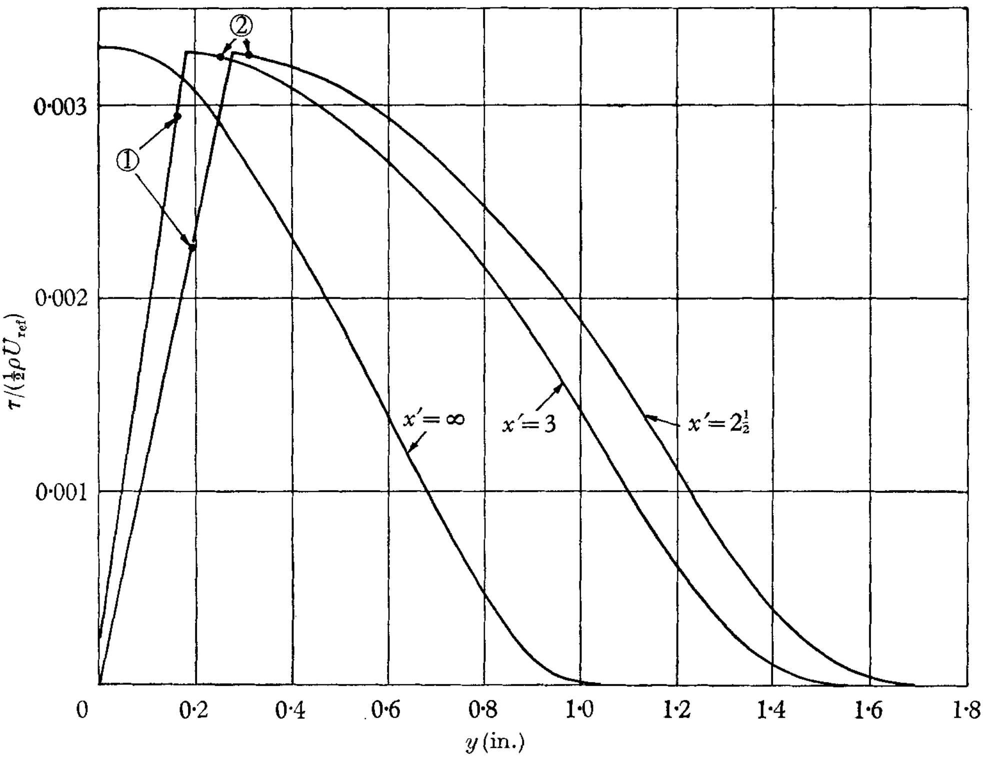


FIGURE 5. Composite shear-stress profiles, using calculated gradients near the surface and Klebanoff's data in the outer region. Points ① indicate departure from square-root dependence of the velocity profiles (see figure 4(c)). Points ② indicate the approximate limit of the effect stress gradients at $\psi = 0{\cdot}05$ (see figure 8(b)).

of the layer to be appreciably augmented by the increased rate of turbulence production resulting from the increased velocity gradients. In fact the increase in velocity gradient is appreciable only in the outermost parts of the layer (figure 4(a)) where the shear stress is small anyway. Near the wall the velocity gradient is *reduced.* In longer regions of adverse pressure gradient, the increased stresses in the outer part of the layer cause an increased transfer of momentum towards the surface and thus delay separation. Therefore, the more rapid the separation the lower is the pressure rise; the extreme case of this being separation from a rearward-facing step and the antithesis being a retarded equilibrium boundary layer, which can be reduced to rest

The velocity profile near the surface has a logarithmic region whose size diminishes as separation is approached. Mellor (1966) has calculated profiles for large values of the pressure-gradient parameter $(\nu/\rho u_\tau^3)(\partial\tau/\partial y)$ (which is less than or equal to $(\nu/\rho u_\tau^3)(dp/dx)$) by using the 'mixing-length' relationship $\tau = \rho K^2 y^2 (\partial u/\partial y)^2$ in the fully turbulent region $u_\tau y/\nu < 30$, the effective viscosity for $u_\tau y/\nu < 30$ being chosen to reproduce the experimentally observed profile in zero pressure gradient. Mellor's profile for $(\nu/\rho u_\tau^3)(\partial\tau/\partial y) = 0{\cdot}1$ is shown in

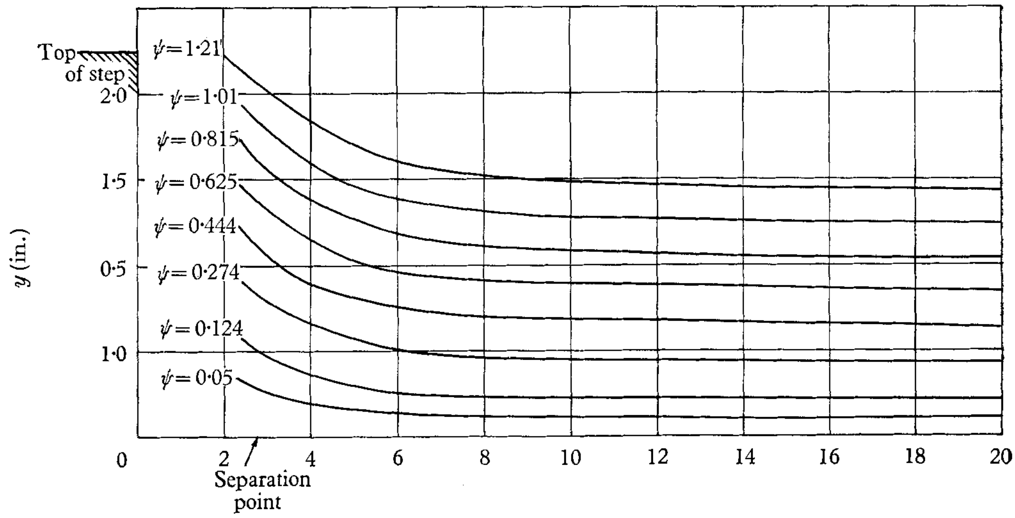


FIGURE 6. Streamlines. $\psi = 1{\cdot}21$ gives approximately the total-pressure boundary. $\phi = 0{\cdot}05$ is approximately the equilibrium-layer boundary.

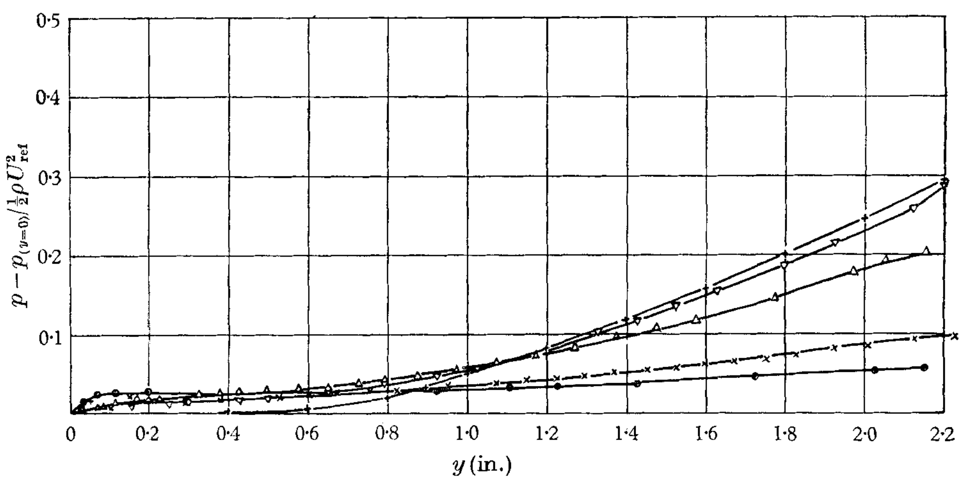
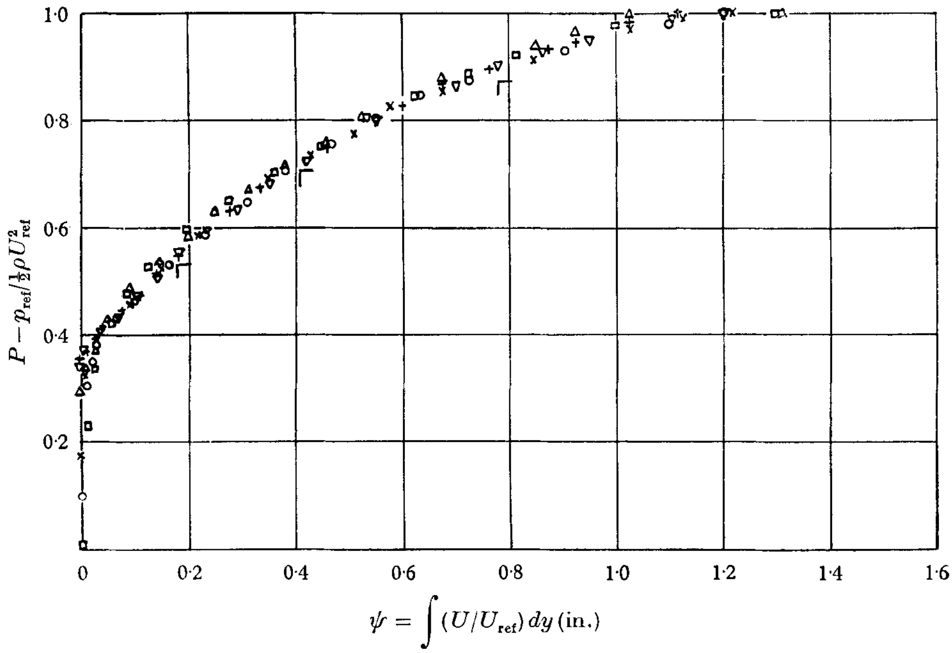
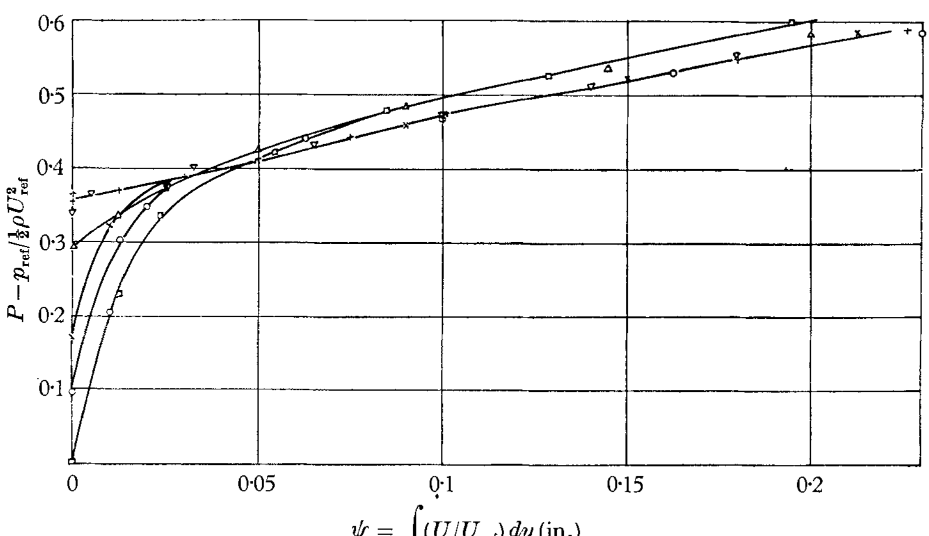


FIGURE 7. Static pressure profiles. $+$, $x' = 2\frac{1}{2}$; ∇, $x' = 3$; $\triangle$, $x' = 4$; $\times$, $x' = 6$; $x' = 8$.

figure 4(*b*) for comparison with the measurements at $x = 4$ in. where $(\nu/\rho u_\tau^3)(\partial\tau/\partial y)$ was also about 0·1 (taking $\partial\tau/\partial y$ to be about half dp/dx). The agreement is fair, bearing in mind the uncertainty of the shear gradient and the inaccuracy of the measurements. The profiles at $x' = 3$ in. and $x' = 2\frac{1}{2}$ in. continue the trend shown by Mellor's calculations but the values of u_τ are untrustworthy. It is usually



(*a*)



(*b*)

FIGURE 8(*a*) Total pressure plotted against stream function; points Γ show the effect of the initial stress gradient (see text). (*b*) Total pressure plotted against stream function near the surface.

	x'	y at $\psi = 0{\cdot}05$
□	∞	0·10
○	8	0·12
×	6	0·14
△	4	0·2
▽	3	0·25
+	$2\frac{1}{2}$	0·31

stated that the logarithmic region vanishes for $(\nu/\rho u_\tau^3)(dp/dx) = 0{\cdot}01 - 0{\cdot}04$; the former value is reached at $x' = 9$ in. and the latter at $x' = 7$ in., whereas the logarithmic region does not shrink to nothing until $x' \simeq 6$ in.; this is merely a confirmation that the criterion should be based on the average value of $\partial\tau/\partial y$ rather than on dp/dx, and that the difference between the two is inevitably large in a rapidly retarded boundary layer because the flow accelerations are large.

The 'mixing length' relationship used by Mellor and the modification of it proposed by Townsend (1961) both predict that

$$U = \frac{2}{K_0}\left(\frac{y}{\rho}\frac{\partial\tau}{\partial y}\right)^{\frac{1}{2}} + U_t,$$

where K_0 is a constant equal to 0·41 (Mellor) or about 0·50 (Townsend), and U_t is a function of τ_w and $\partial\tau/\partial y$, in the outer part of the wall equilibrium layer in a strong adverse pressure gradient. The final departure from this relation occurs at a value of y where the shear-stress gradient starts to decrease appreciably; in the Stratford–Townsend model this point is identified with the inner edge of the 'inviscid' layer. Square-root regions are seen in the velocity profiles (figure 4(*c*)) for $x' = 3$ in. and $x' = 2\frac{1}{2}$ in. only. There is an incipient inflexion in the profile at $x' = 4$ in., but it cannot be distinguished from the experimental scatter. The condition for a square-root region to exist is that the maximum shear stress in the layer shall be very large compared with the surface shear stress, whereas the ratio of the surface shear stress in the initial boundary layer to the surface shear stress at $x' = 4$ in. is only about 4·4. More extensive square-root regions appear when a boundary layer is retarded more slowly so that the shear stress in the central region of the layer has time to rise appreciably above the initial surface shear stress. The additive constant U_t decreases to a very small value at $x' = 2{\cdot}5$ in., slightly *downstream* of the position of separation as indicated by the symmetrical surface tube. Mellor's theoretical value for U_t at separation is about $1{\cdot}2\{(\nu/\rho)(\partial\tau/\partial y)\}^{\frac{1}{3}}$ or about $0{\cdot}008U_{\mathrm{ref}}$; the experimental value at $x' = 2{\cdot}5$ in. is very roughly $0{\cdot}02\,U_{\mathrm{ref}}$, but a value of $0{\cdot}008U_{\mathrm{ref}}$ or less would be well within the experimental accuracy. The values of $\partial\tau/\partial y$ at $x' = 3$ in. and $x' = 2{\cdot}5$ in., calculated from the slopes of the velocity profiles by taking $K_0 = 0{\cdot}5$, are shown in figure 9; it is seen that $\partial\tau/\partial y$ is significantly less than dp/dx even at separation, but the difference between the two is more likely to be negligible in the less rapid separation from an aerofoil, where the flow angles are at least a factor of two less than in the present case. The normal pressure gradient near the surface at $x' = 2\frac{1}{2}$ in. is quite small so that the flow acceleration is predominantly in the x-direction.

The shear-stress profiles, constructed from the measurements of τ_w and $\partial\tau/\partial y$ and Klebanoff's (1955) measurement of the shear-stress profile in a boundary layer in zero pressure gradient (scaled in both co-ordinates) on the assumption that the shear stress in the outer part of the layer is constant along streamlines, are shown in figure 5. The assumption that the join between the inner and outer layers lies on a streamline that passed through the constant-stress layer in the initial boundary layer is seen to be satisfied well enough for practical purposes.

In reality, of course, the shear-stress gradient starts to decrease appreciably at the points (1), and falls to nearly zero at the points (2), so that the join between the two parts of the shear-stress profile is smooth and occurs at a rather smaller shear stress than the join in the idealized profile shown here.

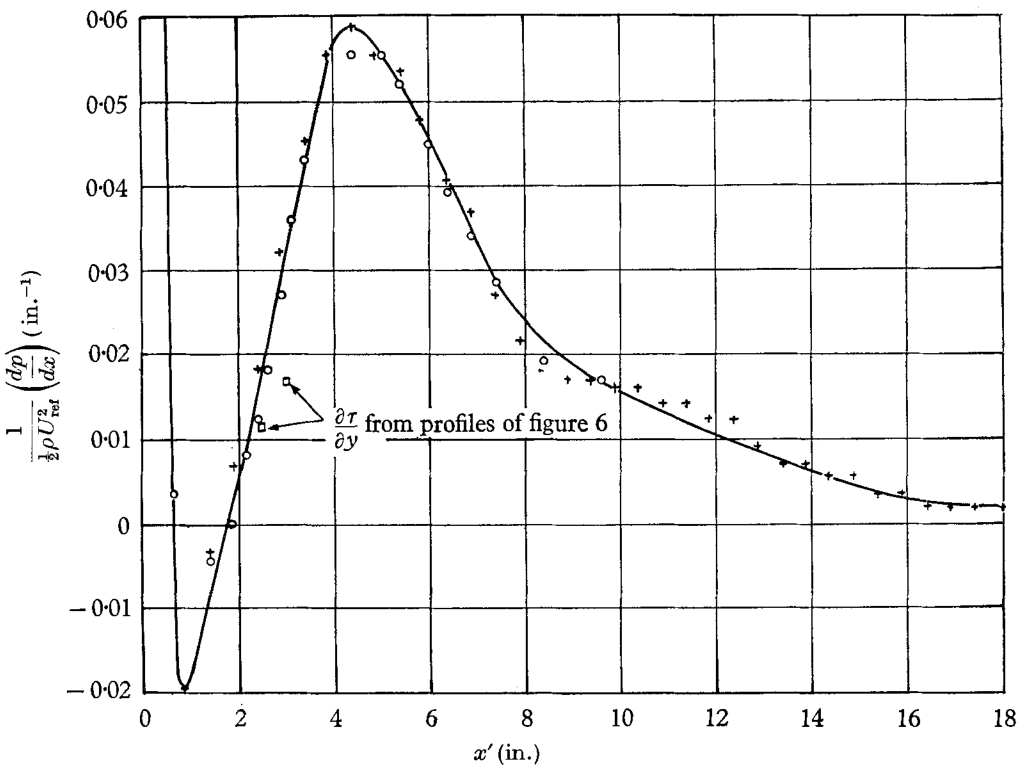


FIGURE 9. Direct measurements of pressure gradient (two runs).

3.3. *The turbulent motion*

The turbulent motion has not been studied in detail because the only region of interest is the wall equilibrium layer, which is rather too thin for useful measurements of shear-stress gradient to be possible. Such measurements would amount to a test of the theoretical square-root profile from which the shear-stress gradients have already been deduced, and this test can be done more conveniently in a thicker retarded boundary layer. The u-component r.m.s. intensity (figure 10) on a given streamline in the outer part of the layer rises slightly as the separation point is approached, but, judging from the frequency spectra (figure 11), the increase is due to low-frequency unsteadiness resulting from modulation of the displacement thickness and the separation position by the turbulent eddies. Kistler (1964) has found appreciable increases in the low-frequency part of the spectrum of the surface pressure fluctuations near a separation point which he ascribed to the same cause. He observed that the fluctuations were not correlated right across the span of the step, and therefore rejected the possibility of any acoustic resonance or other instability of the flow.

The maximum u-component r.m.s. intensity at $x' = 2\frac{1}{2}$ in. occurs at about 0·4 in. from the surface and is only 0·09 of the reference velocity; nearer the surface, the rate of production of turbulent energy is less than in zero pressure gradient because both the mean velocity gradient and the shear stress are less. Therefore, very high intensities of turbulence (referred to the initial free-stream velocity, say) do *not* occur in step-induced separation although the turbulent fluctuations reach a high proportion of the local mean velocity near the separation points, and there is enough low-frequency unsteadiness to make reading a manometer difficult. The high intensities found in less rapidly retarded layers result from increased shear stress and turbulence production in the central part of the layer, and we have already seen that such increased shear stresses do not occur in the present case.

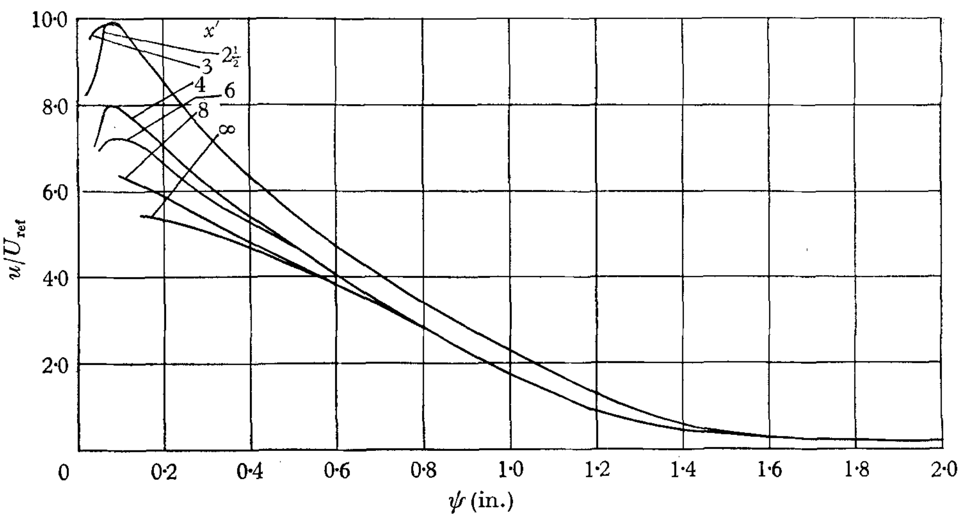


FIGURE 10. R.m.s. u-component intensity plotted against stream function.

4. Methods for the prediction of separation

Townsend (1962) has proposed as a criterion of separation a unique relation between $(\nu/\tau_0^{\frac{3}{2}})(dp/dx)$ and $2K^2(p-p_0)/\tau_0$, where p_0 and τ_0 are the values in the initial boundary layer. Like the rest of the Stratford–Townsend work, this relation applies only to rapid retardation following a region of constant pressure, but an analogous relation could be derived for any known initial state. 'Rapid retardation' means that the stress gradient near the wall, which lies somewhere between $\frac{1}{2}dp/dx$ and dp/dx in most cases, shall be large compared with the initial stress gradients in the outer layer, which are at most slightly larger than τ_0/δ. This is a conservative requirement because the stress gradients in the outer layer are reduced by thickening of the boundary layer. $\partial p/\partial x \gg \tau_0/\delta$ may be written as

$$\frac{\nu}{\tau_0^{\frac{3}{2}}}\frac{dp}{dx} \gg \frac{\nu}{u_\tau \delta},$$

or

$$\log_e \frac{\nu}{\tau_0^{\frac{3}{2}}}\frac{dp}{dx} \gg -6\cdot 7$$

in the present case. (Here and in the rest of the discussion of Townsend's work we follow his use of kinematic quantities.) As seen in figure 12, the condition is well satisfied, except, of course, for large x', the inequality amounting to a factor of 20 in the region of maximum pressure gradient.

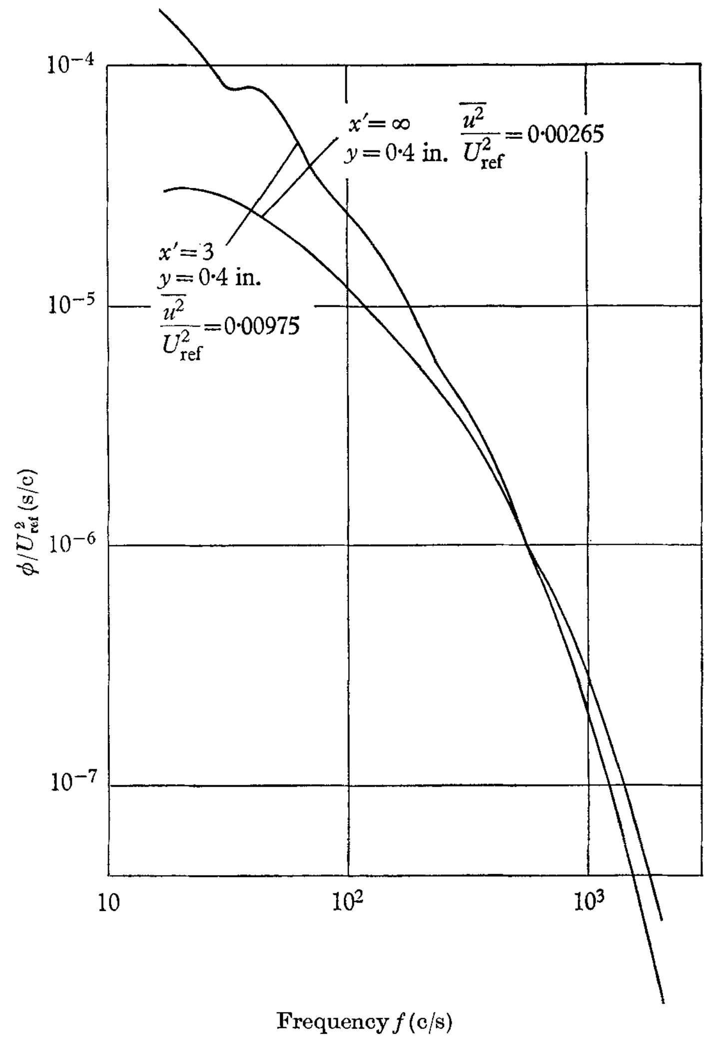


FIGURE 11. u-component frequency spectra: $\int_0^\infty \phi \, df = \overline{u^2}$.

Townsend's separation criterion is also shown in figure 12 and predicts separation at about $x' = 2{\cdot}25$ in., compared with $x' = 2{\cdot}6$–$2{\cdot}7$ in. from the surface-tube measurements. Townsend's criterion agrees much better with the measurements of Schubauer & Klebanoff (conducted at much higher Reynolds number and with less rapid retardation—a factor of only five in the inequality above—and, for both reasons, with a higher value of $(p-p_0)/\tau_0$), but it is difficult to see any reason why the results should be poorer in the present case, except that the criterion depends on the equality of shear stress and pressure gradient at separation so that the ordinate of figure 12 should really be $(\nu/\tau_0^{\frac{3}{2}})(\partial\tau/\partial y)$. If the experimental values of $\partial\tau/\partial y$ are used to replot the points at $x' = 3$ in. and $x' = 2\frac{1}{2}$ in. they are nearer the line which indicates separation, but the apparent trend is such that the predicted separation point would still be well downstream of the actual position. The reduction of pressure gradient prior to the *theoretical* separation point is roughly the factor of five suggested by Townsend but the reduction prior to the actual separation point is only about 2·7:1. However, a more reasonable view to take is that Townsend's criterion predicts the separation point to within about 5% of the length of the region of strong adverse pressure gradient, which is a highly satisfactory result.

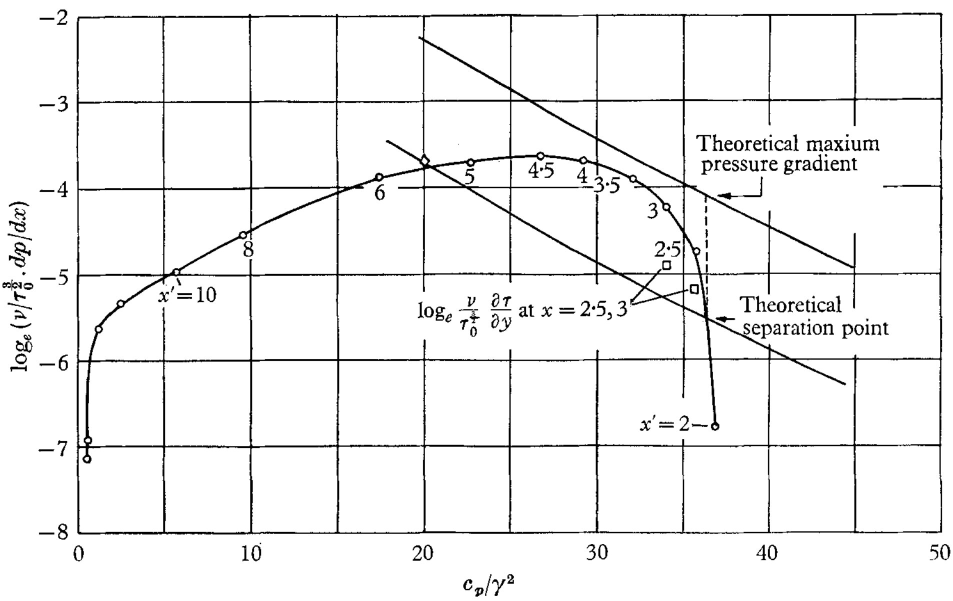


FIGURE 12. Townsend's separation criterion.

Since it is in most cases desired to *avoid* separation rather than to predict it, it would be very useful to have a criterion of imminent or inevitable separation. Townsend (1962) suggests that the final descent to separation starts when the effect of a reduction in pressure gradient (strictly *stress* gradient) is to increase the contribution of the equilibrium layer to the displacement thickness, thus causing a further relief of pressure gradient and a further increase in thickness until the boundary layer separates. The contribution of the equilibrium layer to δ_1 may be written as

$$\Delta\delta_1 = \int_0^{y_s} (1 - U/U_1)\,dy = y_s - \psi_s/U_1,$$

where $y_s = (\tau_0 - \tau_w)/\alpha$, suffix s denoting the edge of the equilibrium layer. Now a change in the stress gradient α does not produce an instantaneous increase in ψ_s—the edge of the equilibrium layer does not jump from one streamline to another—so

$$\frac{\partial}{\partial\alpha}\Delta\delta_1 = \frac{\partial}{\partial\alpha}y_s = -(\tau_0 - \tau_w)\frac{1}{\alpha^2} - \frac{1}{\alpha}\frac{\partial\tau_w}{\partial\alpha},$$

or, putting $\tau_w = t^2\tau_0$,
$$\frac{\alpha}{\tau_0}\frac{\partial}{\partial\alpha}\Delta\delta_1 = -2t\alpha\frac{\partial t}{\partial x} - (1-t^2),$$
where $\partial t/\partial\alpha$ is evaluated at constant U_1. Now $(\partial t/\partial\alpha)_{u_1}$ can be obtained from figure 5 of Townsend (1962) by differentiating an analytical fit to the curves of c_p/γ^2 against t for different values of $\log_e \tau_0^{\frac{3}{2}}/\alpha\nu$ which are given in that figure. We choose
$$c_p/\gamma^2 = \{2{\cdot}37 - 0{\cdot}165(5t-1)^2\}\{\log_e \tau_0^{\frac{3}{2}}/\alpha\nu\}^{\frac{5}{3}},$$
where $\gamma = \tau_0^{\frac{1}{2}}/KU_0$. On substituting values of $\partial t/\partial\alpha$ so obtained into the expression for $\partial(\Delta\delta_1)/\partial\alpha$, we find that the latter is negative *nearly everywhere* in the (t, c_p) plane except for t very near unity (very near the start of the adverse pressure gradient) or $t < 0{\cdot}2$ ($\tau_w < 0{\cdot}04\tau_0$ which is virtually the separation point). Thus the layer will start the final and irrevocable descent to separation only when the 'instantaneous' increase in displacement thickness, caused by the rapid response of the equilibrium layer to a decrease in pressure gradient, outweighs the decrease in *rate* of growth of displacement thickness which is the response of the layer as a whole. It is difficult to derive any useful criterion based on this.

This suggestion of Townsend's has been explored and discussed at some length because he used it to reinforce an argument that the pressure rise to separation could be related to the 'inviscid' pressure gradient just upstream of separation. We now derive what amounts to such a relation by using the Stratford–Townsend model with the sole additional assumption that the edge of the equilibrium layer remains on the same streamline. This enables us to relate the conditions at a station fairly near separation to the pressure rise between that station and the separation point itself. The practical use of this relation would be in cases where a conventional calculation method had been used to predict the development of the boundary layer up to a station where the dimensionless pressure gradient $(\delta_1/\tau_w)(dp/dx)$, or the form parameter H were increasing fairly rapidly so that the conventional method was becoming unreliable. The relation would immediately indicate what additional (rapid) pressure rise could be withstood, and whether separation would occur before the trailing edge of the aerofoil or the end of the diffuser. The *position* of separation could not be predicted very accurately by using the relation, because the 'inviscid' pressure distribution is modified if separation occurs. Clearly the approximation that the equilibrium layer spreads no further into the outer flow means that the additional pressure rise must be even more rapid than that required by the main theory, but it is probably sufficient that the boundary layer at the upstream station shall appear to be in some danger of separation. Denoting conditions on the dividing streamline at the upstream station by suffix s and at separation by suffix ss, we have
$$U_{ss} = \frac{2}{K_0}\left(\frac{\alpha_{ss}y_{ss}}{\rho}\right)^{\frac{1}{2}},$$
neglecting the small additive constant. $\alpha_{ss}y_{ss} = \tau_s$ by conservation of shear stress.

Therefore
$$U_{ss}^2 = 4\tau_s/K_0^2\rho = U_s^2 - 2\Delta p/\rho$$
by Bernoulli's equation, if y_s and y_{ss} lie on the same streamline; Δp is the pressure rise between the upstream station and separation.

Finally, we have
$$2\Delta p/\rho = U_s^2 - 4\tau_s/K_0^2\rho.$$

The quantity on the right-hand side, evaluated at the edge of the equilibrium layer, represents the sudden pressure rise that the boundary layer can withstand. Because in practice the equilibrium layer continues to spread outwards, the figure is a conservative one. Except very near separation, the first term is much the larger so that the values of τ_s need not be very accurate. In the present experiment we do not have a very accurate idea of the position of the edge of the equilibrium layer, but it seems as if the assumption of the theory, that the edge remains on a given streamline, is satisfied over most of the retarded region, so we may take it as being at $\psi = 0{\cdot}05$ in. We can now calculate the overall pressure rise to separation from data at any station as

$$(c_p)_{\max} = c_p + (U_s/U_0)^2 - 4\tau_0/(K_0^2\rho U_0^2).$$

The value of $c_{p_{\max}}$, which is the same at each station since Bernoulli's equation holds on the streamline $\psi = 0{\cdot}05$ in., is 0·40 compared with the actual value of about 0·35. This is a tolerable value and would be improved if we used the actual values of y_s: for $x' = 3$ in., obtaining y_s from figure 4 (*d*) we get

$$(c_p)_{\max} = 0{\cdot}375.$$

Alternatively, a smaller value of ψ_s could be justified by the data in figure 8 (*b*). $\psi_s = 0{\cdot}05$ in. was chosen independently.

A comparison with Schubauer & Klebanoff's results, deducing plausible values of y_s from the square-root region in their measured velocity profiles and reducing their measured values of τ by the factor of 1·5 suggested by Townsend, gives the *extra* pressure rise to separation (c_p at separation $\simeq$ 0·52) shown in table 1.

x(ft.)	20	22·5	24·5	25·4
y_s (in.)	0·2*	0·3	1·5	2·0
Δc_p (calculated)	0·31	0·20	0·07	0·04
Δc_p (actual)	0·35	0·17	0·05	0·01

* Estimated from measured shear stress profile.

Table 1.

The need for a reasonably accurate value of y_s means that the method used to calculate the early development of the boundary layer must be one which predicts the actual velocity or shear-stress profiles. Most methods do this in principle because they are based on a one-parameter family of profiles, but there is a need for improved methods based on modern ideas of the flow in the boundary layer (see Bradshaw *et al.* 1966).

The effect of Reynolds number on the pressure rise for a boundary layer with a given value of the velocity-defect profile parameter G can be calculated on the assumption that y_s/δ is independent of Reynolds number, which should be very nearly true if y_s is outside the viscous sublayer. Taking

$$U/U_1 = (y/\delta)^{1/n}$$

(a more sophisticated profile family could be used), we have

$$\frac{2}{n} = \frac{G\sqrt{(\frac{1}{2}c_f)}}{1-G\sqrt{(\frac{1}{2}c_f)}},$$

and
$$\frac{\Delta p}{\frac{1}{2}\rho U_1^2} = \left(\frac{y_s}{\delta}\right)^{G\sqrt{(\frac{1}{2}c_f)}/\{1-G\sqrt{(\frac{1}{2}c_f)}\}} - \frac{4}{K_0^2}(\tfrac{1}{2}c_f).$$

If we take $G = 6{\cdot}8$ (the value in zero pressure gradient) and $y_s/\delta = 0{\cdot}08$, $\Delta p/\frac{1}{2}\rho U_1^2$ changes from 0·355 at $c_f = 0{\cdot}0033$ (as in the present experiment) to 0·450 at $c_f = 0{\cdot}0023$ (as in Schubauer & Klebanoff's experiment). Barnes (1965) found that the pressure distributions ahead of a normal fence were closely identical, for given δ_1/h, in two tests for which the Reynolds number differed by a factor of 1·5. Since this represents barely 10 % change in c_f the calculations above suggest that the pressure coefficient at separation would change by only about 0·03.

Recently, Goldschmied (1965) has proposed a separation criterion which has some affinity with the one suggested here but which seems less soundly based because it rests on a very questionable assumption about the total pressure at the edge of the viscous sublayer.

Since this paper was submitted for publication we have seen the thesis of Taulbee (1964) who performed a similar experiment at even lower Reynolds numbers ($U_1\delta_2/\nu = 650$ or 1900 at the start of the pressure rise, the boundary layer in the latter case having been artificially thickened). Taulbee's results are generally similar to ours, but his values for the pressure coefficient at separation are noticeably larger than in the present work, being about 0·5 for conditions comparable to those of figure 2. This pressure rise is nearly as large as that attained by Schubauer & Klebanoff's boundary layer at a very much higher Reynolds number and with a less rapid pressure rise. It does not seem to be very plausible. Taulbee's artificially thickened boundary layer was stated to have a maximum value of shear stress of about $1{\cdot}2\tau_w$ within the layer. The other boundary layer, which was tripped by a spanwise row of saw teeth protruding $\frac{1}{32}$ in. into the flow, was assumed to be in a normal state because its thickness was the same as that of an ideal boundary layer becoming turbulent at the leading edge, but its Reynolds number was so low that it may have retained traces of disturbances originating in the transition region.

5. Conclusions

The effect of the additional stress gradients generated by the adverse pressure gradient in the flow up a step is always confined to within one-eighth of the boundary-layer thickness from the surface; total-pressure changes outside this region are due entirely to the stress gradients existing in the unperturbed boundary layer which are small in comparison. The flow up a step is a good test case for the Stratford–Townsend theory of separation.

The separation point is only 1·2 step heights upstream of the step for

$$\delta_1/h = 0{\cdot}086\ (\delta/h \simeq \tfrac{1}{2}) \quad \text{and} \quad \delta_1 U_{\text{ref}}/\nu = 3100.$$

The increase in turbulent intensity up to separation is small, and attributable to low-frequency unsteadiness of the separation point.

The Townsend criterion predicts the separation point fairly accurately, although agreement is poorer than in the comparison with Schubauer & Klebanoff's experiment. The general assumptions and predictions of the Stratford–Townsend theory are in good agreement with experiment. It is pointed out that the theory could be extended to any initial boundary layer (and not merely a constant-pressure boundary layer) and could then be used to predict the final stages of decline to separation after an *extended* region of adverse pressure gradient (in which a conventional calculation method would be used).

If the boundary layer at a given station is sufficiently near separation for the edge of the equilibrium layer to remain on the same streamline until separation, then the pressure rise between the given station and separation is $\frac{1}{2}\rho(U_s^2 - 4\tau_s/\rho K_0^2)$ where U_s and τ_s are the velocity and shear stress at the edge of the equilibrium layer at the upstream station. This quantity is a simple criterion of imminence of separation; once the development of the boundary layer on an aerofoil has been calculated, by a conventional method, to a station where it appears to be in some danger of separation, it can be seen immediately whether the pressure rise to the trailing edge can, in fact, be surmounted. In the case of flow up a step separation is predicted at a pressure coefficient of 0·38–0·40 instead of 0·35; the chief cause of inaccuracy is the difficulty of identifying the edge of the equilibrium layer. In the case of Schubauer & Klebanoff's aerofoil the pressure coefficient at separation is predicted to about $\pm 0{\cdot}03$.

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